The Analysis of Different Cyclists in Individual and Team Time Trials: Based on the Power Profile

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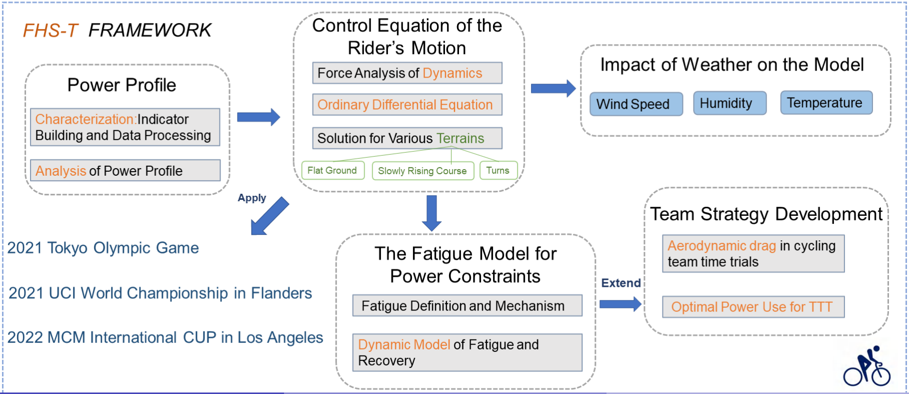
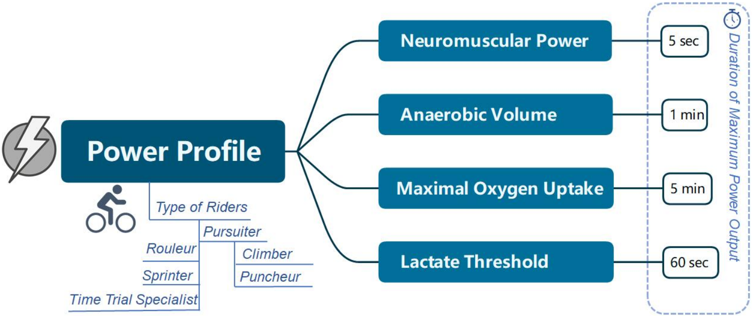
To help riders minimize their cycling time, we need to know the ability of riders and make corresponding strategies. Specifically, we are required to

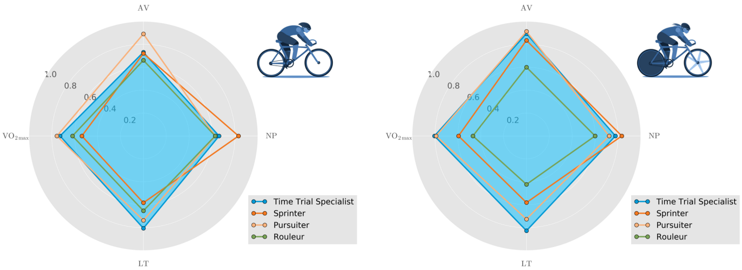
- Create the power profile adapting to various types of riders which can be applied in reality.
- Formulate a plan to optimize the power usage for a team composed of six people in team time trials.



Indicator Building and Data Processing

Depending on the rider's characteristics, each rider's power profile is unique. In this paper, we select gender, weight, and rider types. On one hand, the physiological structure of different genders will be different. On the other hand, the rider's weight will affect the rider's acceleration through the force applied. In addition, facing complex road conditions, different types of riders are specialized in passing different road sections.





(a) Male's Power Profile Diagram (b) Female's Power Profile Diagram

Figure: Power Profile Diagram

If a female rider is skilled in chasing, the anaerobic volume indicator is significantly better. In addition, if the female rider is excellent at sprinting, VO_2max performs remarkably better. This is similarly verified for the power profiles of male riders.

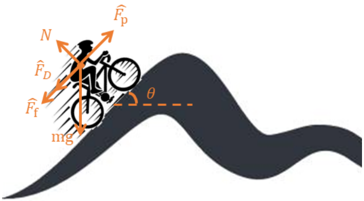


Figure: Force Analysis Diagram

When there is no wind, i.e., $\hat{v}_w$=0,the equation is as follows:

$$m\frac{d^2\hat{x}}{d\hat{t}^2} = \hat{F}_p - \hat{F}_f - \frac{1}{2}\hat{\rho}C_D\hat{A}(\frac{d\hat{x}}{d\hat{t}})^2 - \hat{m}\hat{g}sin\theta(\hat{x}) \quad (1)$$

Use dimensionless methods to elucidate the dependence of the model on the physical parameters:

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{\hat{T}^2\hat{g}}{\hat{L}}(F_\mathrm{p} - F_\mathrm{f}) - \frac{\hat{\rho}c_\mathrm{D}\hat{A}\hat{L}}{2\hat{M}}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 - \frac{m\hat{g}\hat{T}^2}{\hat{L}}\sin\theta(x) \quad (2)$$

We choose to balance the last term with acceleration so that we can take advantage of the last term's smaller advantage later, which gives $\hat{T} = \sqrt{\hat{L}/\hat{g}}$ and $\hat{L} = 2\hat{M}/\hat{\rho} c_{\mathrm{D}} \hat{A}$, where $\hat{M}$ is the average mass of the cycling athletes. Here we consider $F_{\mathrm{p}} = F_{\mathrm{s}}$, where F_{s} is a constant pedal force that continues indefinitely. Later we will consider a complex form. Let $F_0 = F_{\mathrm{s}} - F_{\mathrm{f}}$ to simplify the control equation(2):

$$m\ddot{x} = F_0 - \dot{x}^2 - m \sin\theta(x) \tag{3}$$

The Solution for Flat Ground

we can give an integral equation as follows:

$$\int_{v_0}^{v} \frac{m}{F_0 - \widetilde{v}^2} \mathrm{d}\, \widetilde{v} = t. \tag{4}$$

The solution of this integral equation is:

$$\dot{x}(t) = \begin{cases} \sqrt{F_0} \tanh\left(\xi_0(t)\right) & v_i < \sqrt{F_0} \\ \sqrt{F_0} \coth\left(\bar{\xi}_0(t)\right) & v_i > \sqrt{F_0} \end{cases} \tag{5}$$

$$x(t) = \begin{cases} m \log\cosh\left(\xi_0(t)\right) + \sigma_0 & v_i < \sqrt{F_0} \\ m \log\sinh\left(\bar{\xi}_0(t)\right) + \sigma_0 & v_i > \sqrt{F_0} \end{cases} \tag{6}$$

where $\sigma_0 = m/2 \log\left|v_i^2/F_0 - 1\right|$ and

$$\begin{aligned} \xi_0(t) &= \sqrt{F_0}\, t/m + \operatorname{artanh}\left(v_i/\sqrt{F_0}\right) \\ \bar{\xi}_0(t) &= \sqrt{F_0}\, t/m + \operatorname{artanh}\left(\sqrt{F_0}/v_i\right). \end{aligned}$$

The Solution for Slowly Rising Course

we modify the parameter $\theta(x)$ to obtain the appropriate solutions for weakly undulating course:

$$\theta(x) = \alpha + \epsilon f(x) \tag{7}$$

where α is a mean value of different varies of θ, $0 < \epsilon \ll 1$, and $f = O(1)$, so we can expand the displacement as a power series with ϵ.

$$x = x_0 + \epsilon x_1 + \epsilon^2 x_2 + \cdots = \sum_{i=0}^{\infty} \epsilon^i x_i \tag{8}$$

By substituting (7) and (8) into (4) with initial conditions, we can get

$$O(1): \quad \begin{cases} m\ddot{x}_0 = F_0 - \dot{x}_0^2 - m\sin\alpha, \\ x_0(0) = 0, \quad \dot{x}_0(0) = v_i, \end{cases} \tag{11.a}$$

$$O(\varepsilon): \quad \begin{cases} m\ddot{x}_1 = -2\dot{x}_0\dot{x}_1 - mf(x_0)\cos\alpha, \\ x_1(0) = 0, \quad \dot{x}_1(0) = 0. \end{cases} \tag{11.b}$$

Table: Basic data of Annemiek Van Vleutenon

Date of Birth	8th October 1982 (39)
Nationality	Netherlands
Weight	59 kg
Height	1.68 m

Below are some values of system parameters typically expected in a race:
$\hat{M} = 60kg, F_{\mathrm{f}} = 3N, F_{\mathrm{p}} = 25N, \rho = 1.3\mathrm{kg/m^3}, c_{\mathrm{D}}, \hat{A} = 0.3m^2$.

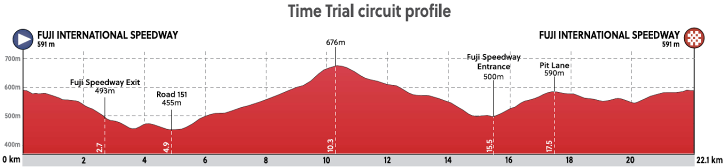
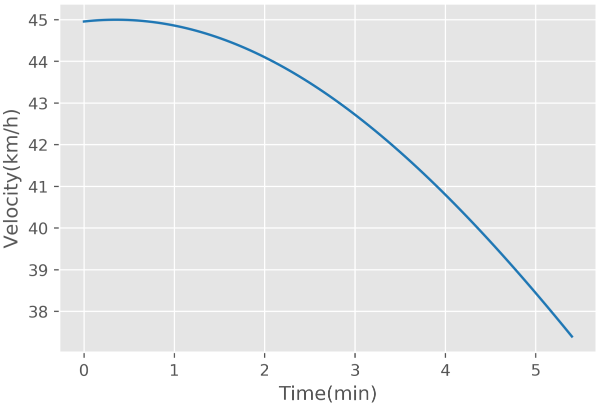


Figure: Time Trial Circuit Profile of 2021 Tokyo Olympic Games

Figure: The Athlete Log in 2021 Tokyo Olympic Games

By applying the previous equation solution for the hill, we obtain the velocity profile. Specifically, we introduce $\alpha = 2.3^\circ$ and values of related parameters to 11.b. Then we can get

$$v(t) = \ddot{x}(t) = 45\mathrm{sech}^2(0.0866t + 0.0308)$$



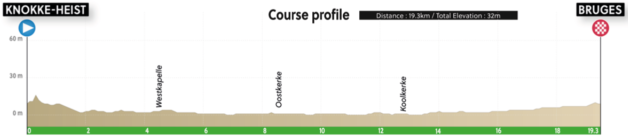


Figure: Time Trial Circuit Profile of 2021 Road World Championships

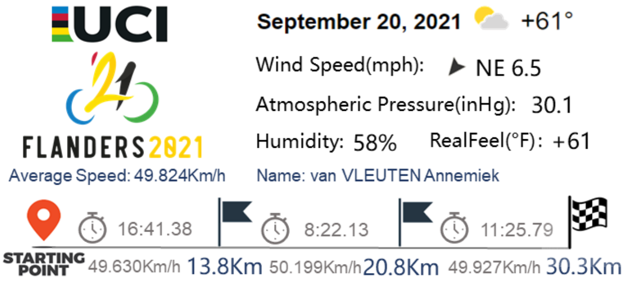


Figure: The Athlete Log in 2021 Road World Championships

$$30\sqrt{3}\tanh(0.866t + 0.0308)$$

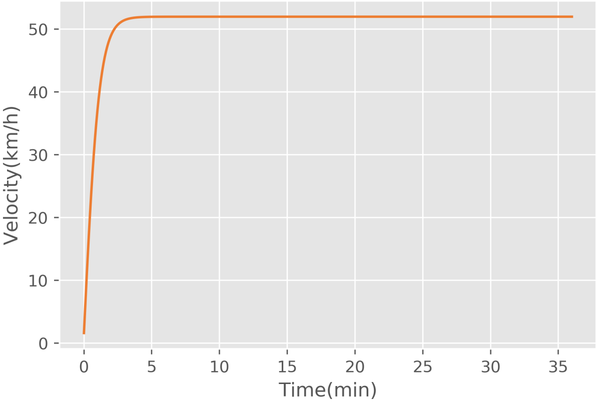


Figure: The Velocity Power Curve for Flanders

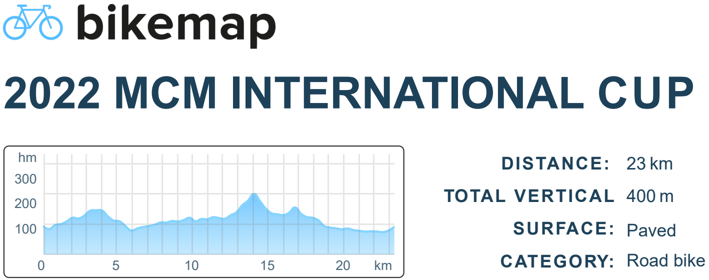


Figure: 2022 MCM International CUP

The course starts at 506 Yale St, Los Angeles, CA 90012 (34°03'40.7"N 118°14'30.3"W) and finishes at 587-599 Ord St, Los Angeles, CA 90012 (34°03'40.2"N 118°14'30.0"W). The two types of terrain covered by this course have been demonstrated separately in the above races, so the specifics can be found in the analysis of the first two courses.

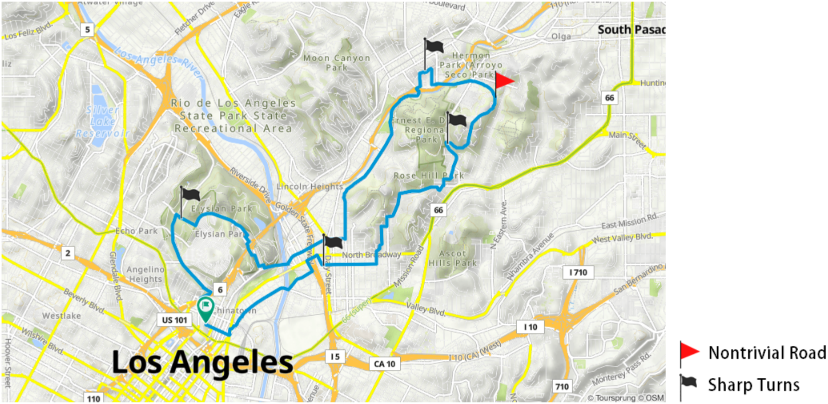


Figure: 2022 MCM International CUP Map

We set upwind speeds of 4.5 m/s, 4.75 m/s, 5 m/s, 5.25m/s, and 5.5 m/s to analyze the change of speed within 4.8 minutes (the situation is similar for downwind).

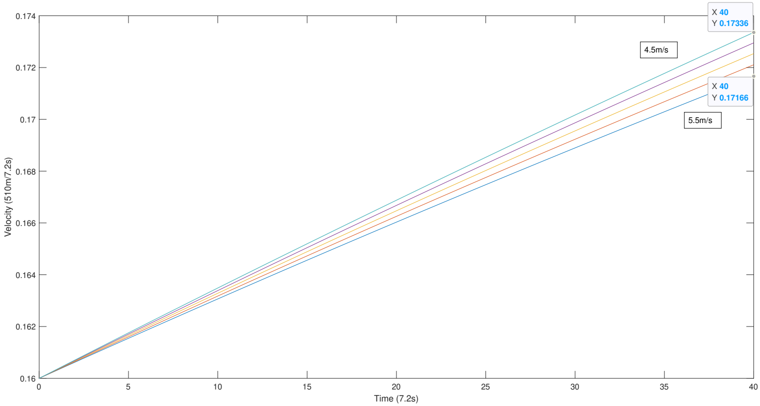
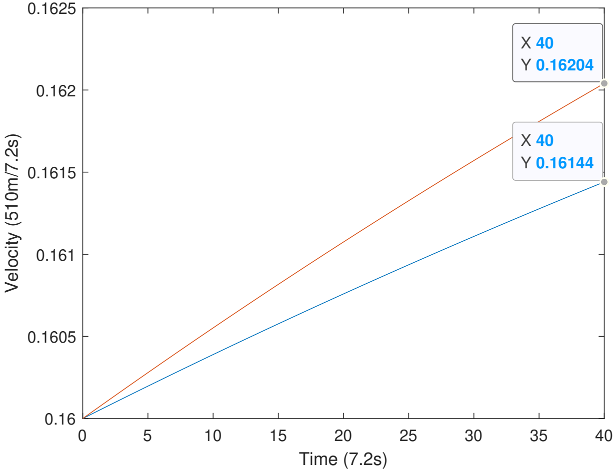


Figure: Sensitivity Analysis for the Wind Speed

Humidity has a large impact on the coefficient of friction between tires and the ground, and there is a potential threat of tire slippage. We assume that the friction coefficient changes from 0.3 to 0.2 in the windless condition. F_0 will change its value accordingly, thus affecting the speed.



Thanks!